The Style Drift Score

A quantitative measure.

Thomas M. Idzorek and Fred Bertsch

Style drift is the evolution of a portfolio's investment style over time, where style is determined using return-based style analysis (see Sharpe [1988, 1992]). Style drift can produce unwanted and unexpected style bets for individual portfolios and fund of funds portfolios.

The development of sophisticated software has brought return-based style analysis to a large number of investors (see Hardy [2003]). Practitioners can gain a qualitative feeling for the style consistency of a portfolio from rolling-window asset allocation graphs and style maps, which illustrate the evolution of a portfolio's investment style. These are excellent tools for developing an intuitive understanding of a portfolio's style consistency, but they do not give us a quantitative measure of style drift.

A quantitative measure of style drift provides a new statistic to screen the ever-growing number of investment managers, to compare the style consistency of multiple managers, and to monitor the drift in a manager's style. We review possible measures of style drift that we find inadequate. We then derive our *style drift score,* apply it to several portfolios, and present some empirical observations for a large sample of mutual funds. We show how the style drift score can be used to monitor the style drift of a portfolio.

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RETURN-BASED STYLE ANALYSIS

Sharpe [1988, 1992] demonstrates that it is possible to determine a portfolio's "effective asset mix" using the historical returns of broad asset classes (proxied by available market indexes). More formally, return-based style analysis takes the form:

$$r_{p,t} = \left[c_1 a_{1,t} + c_2 a_{2,t} + \cdots + c_K a_{K,t}\right] + e_t \qquad (1)$$

where

$r_{p,t}$ is the return of the portfolio for $t = 1, 2, \ldots, T$;

$c_1, \ldots, c_K$ are the asset class coefficients for $k = 1, 2, \ldots, K$;

$a_{1,t}, \ldots, a_{K,t}$ are the period t returns for the K asset classes; and

e_t is the excess return at time t (i.e., the portion of the return that is not explained by the return of the K asset classes).

Equation (1) can be rearranged as follows:

$$e_t = r_{p,t} - \left[c_1 a_{1,t} + c_2 a_{2,t} + \cdots + c_K a_{K,t}\right] \qquad (2)$$

In this case, e_t is the excess return (positive or negative) at time t relative to the returns of the appropriately weighted asset classes. The excess return is the difference between the return of the portfolio at time t ($r_{p,t}$) and the return of the K asset classes at time t ($a_{1,t}, \ldots, a_{K,t}$) weighted by the K asset class coefficients ($c_1, \ldots, c_K$). While we have chosen to use the term, *coefficient*, one can substitute beta, allocation, weight, or factor loading.

Sharpe's method determines the asset class coefficients ($c_1, \ldots, c_K$) that minimize the variance of the excess return series (e_t), typically subject to $c_k \geq 0$ for $k = 1, 2, \ldots, K$, and $c_1 + c_2 + \ldots + c_K = 1$. In other words, the values of the individual coefficients, or exposures, to the K asset classes are equal to or greater than 0 and collectively sum to 1.

A common misconception regarding return-based style analysis is that it minimizes the sum of the squared differences between the return of the manager and the weighted return of the K asset classes. For a detailed discussion of this misconception and the mathematical basis of return-based style analysis, see Becker [2003].

STYLE DRIFT

A portfolio's effective asset mix is the value of the asset class coefficients ($c_1, \ldots, c_K$), and the style drift of a portfolio is the evolution of the asset class coefficients over time.

In general, the more data points used to calculate the asset class coefficients, the smoother the changes. With each new data point, the asset class coefficients are updated, which normally results in small changes in the asset class coefficients. In practice, the degree of change in the asset class coefficients from one period to the next is a function of the number of periods of data in the calculation and how much the portfolio's style changed.

Evolution of the asset class coefficients (that is, the portfolio's effective asset mix) can be shown in a rolling- window asset allocation graph (such as graphs at the left in Exhibit 1). In this case, a fixed number of time periods, such as 36 months, are used to compute the asset class coefficients. When a new data point enters the calculation, the oldest data point is dropped, so that the number of time periods used to calculate the asset class coefficients remains fixed. Changes in the asset class coefficients are called *style drift*.

Style drift can also be visualized using rolling-window style maps. The most common style map plots portfolios using a value-growth score and a small-large score (as in the graphs at the right in Exhibit 1). The larger symbols represent more recent data points.[1]

The 36-month rolling-window asset allocation graphs and style maps in Exhibit 1 indicate that Portfolio 2 has more style drift than Portfolio 1.

POSSIBLE MEASURES OF STYLE DRIFT

Possible measures of style drift are tracking error relative to a market benchmark or custom style benchmark, and style benchmark turnover.

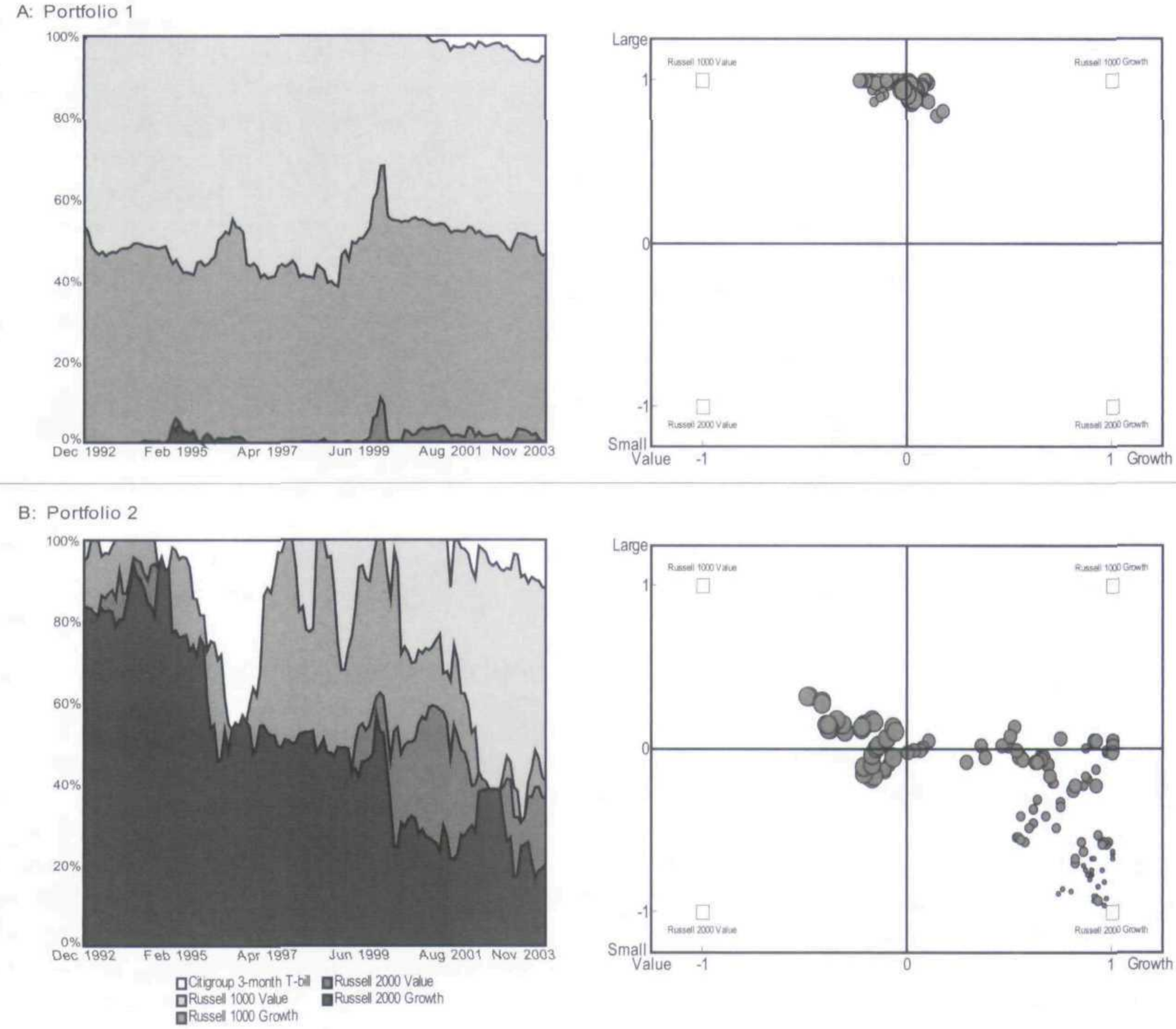
Tracking Error

Tracking error relative to a market benchmark can be a reasonable measure of style consistency.[2] If tracking error relative to the market benchmark is low, the style of the portfolio is probably consistent with that of the market benchmark. If tracking error relative to the market benchmark is high, the portfolio's style consistency is unknown. High tracking error can result from a stable set of style bets (style consistency) away from the market benchmark or from a set of changing style bets (style drift).

Tracking error relative to a style benchmark is an excellent measure of *pure active risk* (the standard deviation of active returns after adjusting for market exposures; see Siegel [2003]). Tracking error in this case does not measure style drift directly. The style benchmark is the return series produced by weighting the asset class returns ($a_{1,t}, \ldots, a_{K,t}$) by the asset class coefficients ($c_1, \ldots, c_K$). The asset class coefficients behind the style benchmark evolve over time, so tracking error relative to a style benchmark does not measure the style drift of a portfolio. It is the evolu-

Exhibit 1

Rolling Asset Allocation Graphs and Style Maps



tion of the asset class coefficients over time that indicates style drift.

Style Benchmark Turnover

Style benchmark turnover is the change in the style benchmark from one rolling period to the next, after adjusting the asset class coefficients for natural drift. Style benchmark turnover is an inadequate measure of style drift because it fails to differentiate between a portfolio that moves from one style to another and a portfolio that oscillates within a relatively narrow style range.[3]

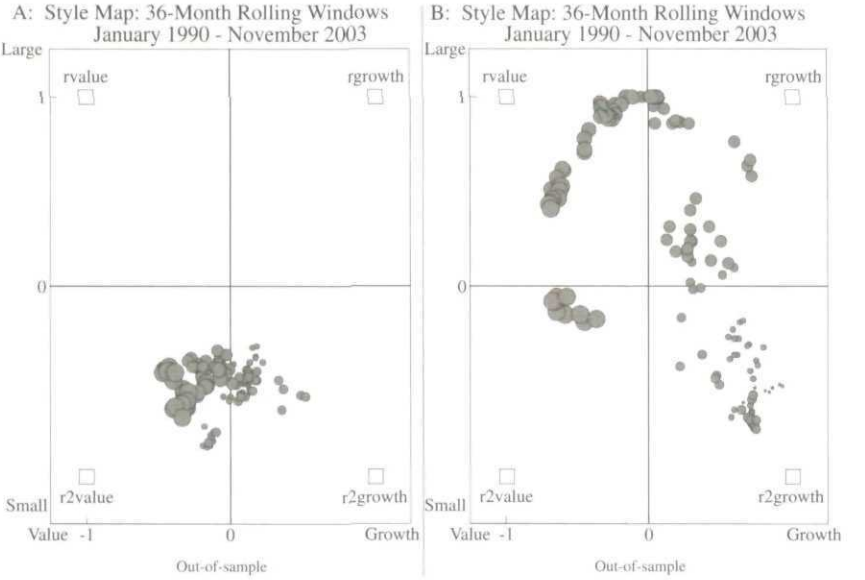
Two managers with similar style benchmark turnover numbers (63 and 64) are plotted on a style map in Exhibit 2. The style of the manager depicted in Panel A oscillates in an area of the style map that is predominately small core. The style of the manager in Panel B has been small growth, mid-growth, large growth, large core, and large value, and is currently closest to mid-value. In this case, similar style benchmark turnover numbers are not indicative of similar amounts of style drift.

STYLE DRIFT SCORE

The inadequacy of the statistics to measure style drift prompted us to develop a new statistic, the *style drift*

EXHIBIT 2

Managers with Same Style Benchmark Turnover



score, which measures the variability of style through time. The style drift score is the square root of the sum of the variances of the asset class coefficients:

$$SDS = \sqrt{\sum_{k=1}^{K} \sigma_k^2} \tag{3}$$

where $\sigma_k^2 = \mathrm{Var}\,\{c_{k,1}, c_{k,2}, \ldots, c_{k,T}\}$ is the variance of the k-th asset class coefficient.

Measuring changes or differences in sets of coefficients is similar in spirit to the various econometric techniques associated with structural change, such as the Chow test [1960]. These traditional econometric techniques are usually applied to the residuals resulting from a standard linear regression, but return-based style analysis residuals are not the same as the standard linear regression residuals. It is this subtle difference that motivates our new approach.

The style drift score is calculated by measuring the aggregate variability of the individual asset class coefficients:

$$SDS = \sqrt{Var\begin{bmatrix} c_{1,1} \\ \vdots \\ c_{1,T} \end{bmatrix} + Var\begin{bmatrix} c_{2,1} \\ \vdots \\ c_{2,T} \end{bmatrix} + \cdots + Var\begin{bmatrix} c_{K,1} \\ \vdots \\ c_{K,T} \end{bmatrix}} \tag{4}$$

where $c_{k,t}$ is the k-th asset class coefficient at time t.

Equations (3) and (4) demonstrate the simplicity of the calculation, but they mask the underlying intuitiveness. An alternative calculation of the style drift score yields the same answer, but reveals more of the intuition.

Start by calculating the average value of the asset class coefficients for times 1 through T. Let $\bar{c}_1, \bar{c}_2, \ldots, \bar{c}_K$ be the average values of the K asset class coefficients. $\bar{c}_1, \bar{c}_2, \ldots, \bar{c}_K$ can be thought of as the *center of gravity* in K-dimensional space:

$$\begin{aligned} \bar{c}_1 &= Average\{c_{1,1}, c_{1,2}, \cdots, c_{1,T}\} \\ \bar{c}_2 &= Average\{c_{2,1}, c_{2,2}, \cdots, c_{2,T}\} \\ &\vdots \\ \bar{c}_k &= Average\{c_{K,1}, c_{K,2}, \cdots, c_{K,T}\} \end{aligned} \tag{5}$$

Next, subtract the average values of the K asset class coefficients from the respective style coefficients at times 1 through T:

$$\begin{matrix} (c_{1,1} - \bar{c}_1) & (c_{2,1} - \bar{c}_2) & \cdots & (c_{K,1} - \bar{c}_K) \\ (c_{1,2} - \bar{c}_1) & (c_{2,2} - \bar{c}_2) & \cdots & (c_{K,2} - \bar{c}_K) \\ \vdots & \vdots & \vdots & \vdots \\ (c_{1,T} - \bar{c}_1) & (c_{2,T} - \bar{c}_2) & \cdots & (c_{K,T} - \bar{c}_K) \end{matrix} \tag{6}$$

This results in a $K \times T$ matrix of calculations where each row represents the component-wise distance from the center of gravity for that time period. For each time period, the individual K distances are squared, and the sum is calculated, resulting in a $T \times 1$ vector. Each element of the $T \times 1$ vector represents the squared Euclidian distance between the asset class coefficients at time t and the center of gravity:

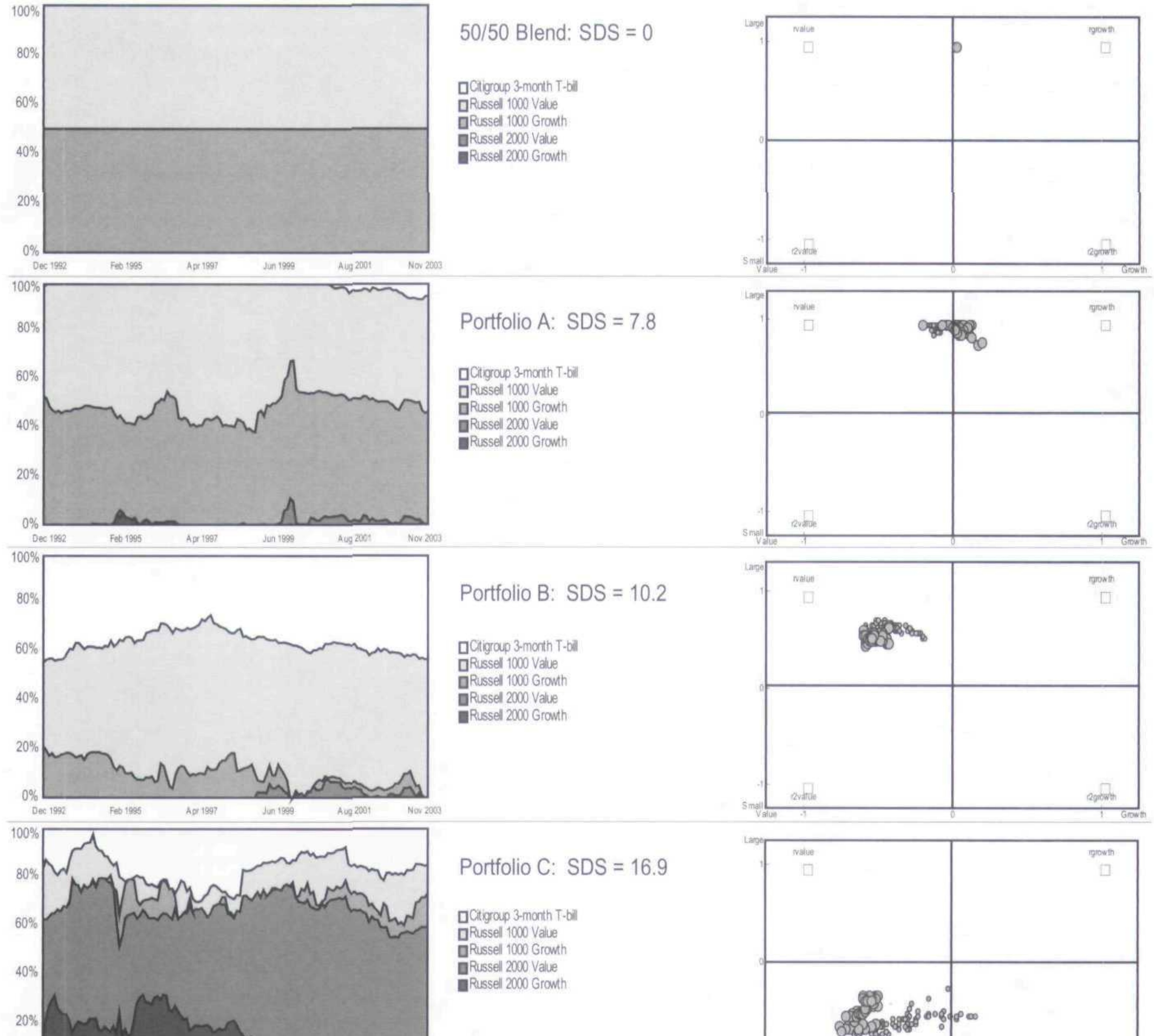
$$\begin{matrix} (c_{1,1} - \bar{c}_1)^2 + (c_{2,1} - \bar{c}_2)^2 + \cdots + (c_{K,1} - \bar{c}_k)^2 = \text{Distance}_1^2 \\ (c_{1,2} - \bar{c}_1)^2 + (c_{2,2} - \bar{c}_2)^2 + \cdots + (c_{K,2} - \bar{c}_k)^2 = \text{Distance}_2^2 \\ \vdots \qquad \vdots \qquad \vdots \qquad \vdots \\ (c_{1,T} - \bar{c}_1)^2 + (c_{2,T} - \bar{c}_2)^2 + \cdots + (c_{K,T} - \bar{c}_T)^2 = \text{Distance}_T^2 \end{matrix} \tag{7}$$

Next, calculate the sample average of the $T \times 1$ vector of squared Euclidian distances from the center of gravity:

$$\text{Average Squared Euclidian Distance} = \frac{\sum_{t=1}^{T} \text{Distance}_t^2}{T-1} \tag{8}$$

EXHIBIT 3

Portfolios with Increasing Style Drift Scores



Finally, calculate the square root of the average squared Euclidian distances:

$$SDS = \sqrt{\text{Average Squared Euclidian Distance}} \tag{9}$$

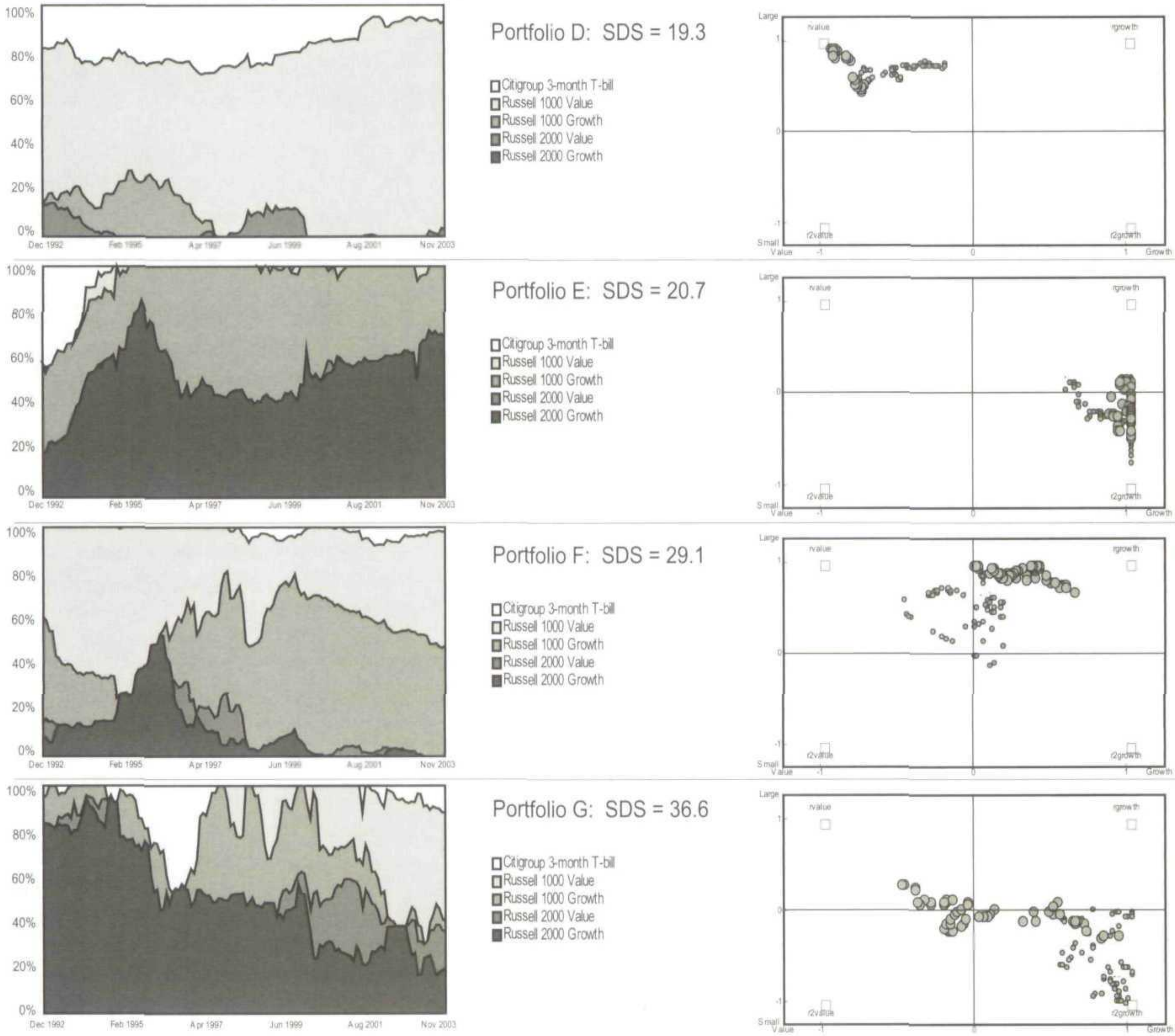
The style drift score is the average Euclidian distance of the T rolling-window asset class coefficients from the center of gravity in K-dimensional space.[4]

Examples

Exhibit 3 presents eight rolling-window asset allocation graphs and style maps ranked by their respective style drift score. The style drift score (SDS) is given in each panel. The first rolling allocation graph and style map represent a blend of the Russell 1000 Growth and Russell 1000 Value indexes, rebalanced monthly. As one would

E X H I B I T 3 (continued)

Portfolios with Increasing Style Drift Scores



expect, the style drift score for this constant style blend is 0, indicating perfect style consistency or no style drift.

Each pair of rolling-window asset allocation graphs and style maps in Exhibit 3 depicts a portfolio with greater style drift than the previous portfolio. As one would expect, the style drift score also increases.

Distribution of the Style Drift Score

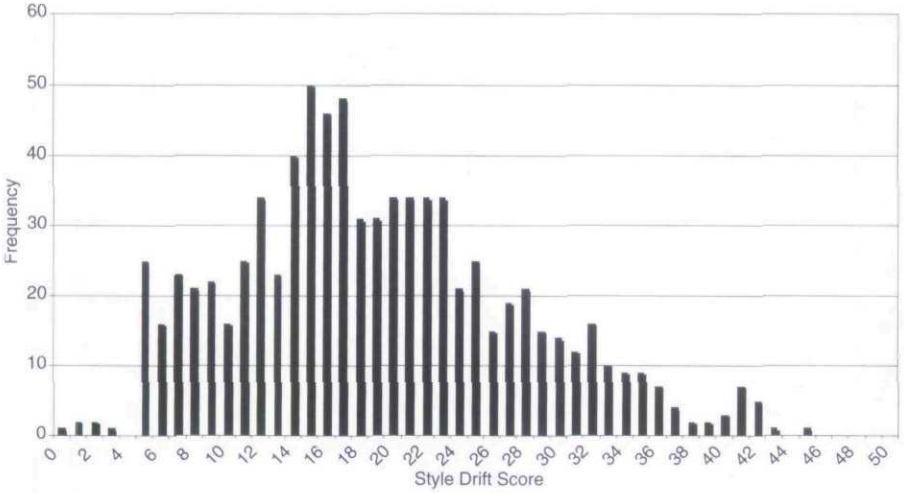
To gain a better feel for the range of style drift scores, we examine a large set of mutual funds in the Morningstar Equity database, conducting a return-based style analysis for all mutual funds with at least ten years of data. Mutual funds with an R^2 below 50 were removed, and we calculated the style drift score for the remaining 812 funds.[5]

Exhibit 4 provides a histogram of the 812 managers. The average style drift score in this case is 18.54.

RECOMMENDATIONS FOR USE

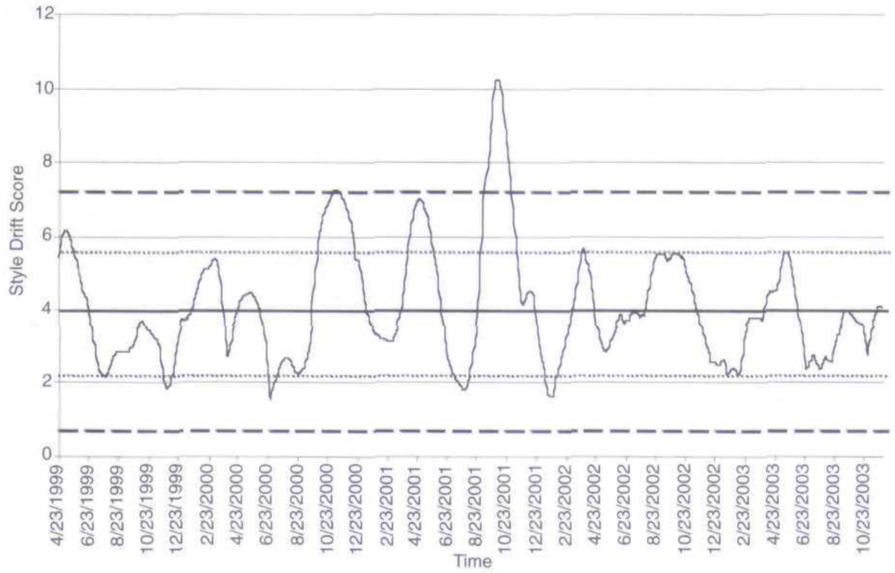
The comparison of style drift scores is most meaningful when the model parameters (style benchmark

EXHIBIT 4
Style Drift Score Distribution—Morningstar Managers



Based on 36-month moving windows, computed monthly October 1993–September 2003.

EXHIBIT 5
Style Drift Monitoring



Based on 60-day moving windows, computed daily April 23, 1999–October 23, 2003.

indexes, objective function, constraints, periodicity, and date range) for the calculations are the same. That is to say, our style drift measure is most useful when the style models compared have exactly the same quantitative specification.

Beyond quantitative screens, the measure can be used to monitor the style consistency of portfolios. We believe style consistency can be indicative of a skillful portfolio manager and a successful risk management system, and is a desirable attribute in managers. It also has obvious benefits in the fund of funds portfolio construction process. In fact, Brown and Harlow [2002] conclude there is a positive relationship between investment style consistency and performance.

As in the Goldman, Sachs "green zone" tracking error methodology (see Litterman et al. [2000]), which monitors the N-period rolling annualized tracking error over time, one can detect abnormal amounts of style drift using the style drift score. This is especially useful for monitoring off-market benchmark managers.[6]

Exhibit 5 graphs the style drift score for a mutual fund based on 60 days of data and basing the center of gravity on the same 60 days. For simplicity, we create zones based on one and two standard deviations from the average style drift score of the mutual fund over the entire time period. In practice, the zones would be established in accordance with the user's particular needs. When the style drift score exceeds a certain zone, this may be cause to investigate.

The first zone (bounded by the dotted lines) represents the expected amount of style drift. The second zone (bounded by the dashed lines) represents possible cause for concern. The third zone (above the top dashed line and below the bottom dashed line) represents definite cause for concern. Such an analysis could be part of a manager scoring and evaluation system.

CONCLUSION

Our style drift score measures the variability of a portfolio's effective asset mix (style) around its average effective asset mix (average style). Low numbers represent low amounts of style drift, and high numbers represent large amounts of style drift.

The main advantage of the style drift score is that it makes evaluation of numerous rolling-window graphs unnecessary by providing a screening statistic to identify a subset of managers for further study.

ENDNOTES

The authors thank Stephen Hardy for supporting this project, Thomas Becker for his mathematical wizardry, and Daniel Coggin for helpful comments.

[1]The coordinates for the style maps at time t are derived using the asset class coefficients $(c_t, \ldots, c_{K,t})$. For the style maps in Exhibit 1, the x coordinate equals $(c_{2,t} + c_{4,t}) - (c_{1,t} + c_{3,t})$ where c_2 = allocation to Russell 1000 Growth, c_4 = allocation to Russell 2000 Growth, c_1 = allocation to Russell 1000 Value, and c_3 = allocation to Russell 2000 Value. The y coordinate is $(c_{1,t} + c_{2,t}) - (c_{3,t} + c_{4,t})$.

[2]Tracking error relative to a market benchmark is the standard deviation of the excess return series derived from subtracting the market benchmark return from the manager's return. Tracking error relative to the style benchmark is the standard deviation of e_t from Equation (2).

[3]Style benchmark turnover (SBT) is:

$$SBT = \frac{\sum_{t=1}^{T}\left(\sum_{k=1}^{K} \left| \frac{c_{k,t}\left(1 + r_{k,t+1}\right)}{\sum_{k=1}^{K} c_{k,t}\left(1 + r_{k,t+1}\right)} - c_{k,t+1} \right| \Bigg/ 2 \right)}{T} \times \text{(Periods per Year)}$$

[4]A useful modification to the style drift score is to substitute a manager's normal portfolio weights (or strategic policy benchmark weights) for the original average values of the K asset class coefficients $(\bar{c}_1, \bar{c}_2, \ldots, \bar{c}_K)$. Essentially, we are redefining the center of gravity. One could update the center of gravity to reflect periodic changes in the target portfolio weights. Such a measure can be used in conjunction with the portfolio's tracking error relative to the strategic policy benchmark, i.e., target portfolio weights. This modified version of the style drift score could be used to compare the degree of asset allocation rotation of portfolios with similar tracking errors. From a performance attribution perspective, it stands to reason that the performance of portfolios with a low score is attributable to security selection while the performance of portfolios with a high score is attributable to tactical asset allocation.

[5]We began with all the funds categorized as large growth, large blend, large value, mid-growth, mid-blend, mid-value, small growth, small blend, and small value.

[6]Off-market benchmark managers, which represent the majority of managers, are managers whose performance should be monitored relative to a blended, multi-index style benchmark that represents their "normal" investment style. A custom style benchmark enables one to determine a manager's true or pure alpha after accounting for various market exposures (see Waring and Siegel [2003]).

REFERENCES

Becker, Thomas. "Exploring the Mathematical Basis of Returns-Based Style Analysis." In T. Daniel Coggin and Frank J. Fabozzi, eds., *The Handbook of Equity Style Management,* 3rd ed. New York: John Wiley & Sons, Inc., 2003.

Brown, Keith C., and W.V. Harlow. "Staying the Course: The Impact of Investment Style Consistency on Mutual Fund Performance." Working paper, March 2002. Available at www.mccombs.utexas.edu/faculty/keith.brown/Research/styleconsistent-wp.pdf.

Chow, Gregory C. "Tests of Equality between Sets of Coefficients in Two Linear Regressions." *Econometrica,* 28 (1960), pp. 591-605.

Hardy, R. Stephen. "Style Analysis: A Ten-Year Retrospective and Commentary." In T. Daniel Coggin and Frank J. Fabozzi, eds., *The Handbook of Equity Style Management,* 3rd ed. New York: John Wiley & Sons, Inc., 2003.

Litterman, Robert, Jacques Longerstaey, Jacob Rosengarten, and Kurt Winkelmann. "The Green Zone: Assessing the Quality of Returns." Goldman, Sachs & Company, March 2000.

Sharpe, William F. "Asset Allocation: Management Style and Performance Measurement." *The Journal of Portfolio Management,* Winter 1992, pp. 7-19.

——. "Determining a Fund's Effective Asset Mix." *Investment Management Review,* December 1988, pp. 59-69.

Siegel, Laurence B. *Benchmarks and Investment Management.* The Research Foundation of the Association for Investment Management and Research, 2003.

Waring, M. Barton, and Laurence B. Siegel. "The Dimensions of Active Management." *The Journal of Portfolio Management,* Spring 2003, pp. 35-52.

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